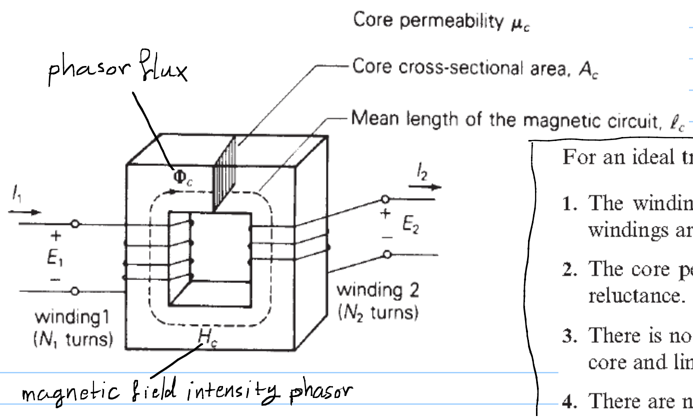
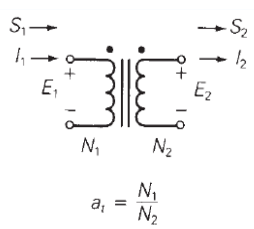
Ch3. Power Transformers

3.1 The Ideal Transformer :



For an ideal transformer, the following are assumed:

1. The windings have zero resistance; therefore, the I^2R losses in the windings are zero.
2. The core permeability μ_c is infinite, which corresponds to zero core reluctance.
3. There is no leakage flux; that is, the entire flux Φ_c is confined to the core and links both windings.
4. There are no core losses.



Amper's Law:

$$\oint H_{\tan}\, dl = I_{\text{enclosed}} \Rightarrow H_c l_c = N_1 I_1 - N_2 I_2$$

The magnetic flux density:

$$B_c = \mu_c H_c \quad \text{Wb/m}^2$$

The core flux:

$$\Phi_c = B_c A_c \quad \text{Wb}$$

$$N_1 I_1 - N_2 I_2 = l_c B_c / \mu_c = \underbrace{\left(\frac{l_c}{\mu_c A_c}\right)}_{R_c} \Phi_c$$

(Core reluctance)

μ_c is assumed $\infty \rightarrow R_c = 0$,

$\therefore N_1 I_1 = N_2 I_2$

Dot Notion:

① If $\rightarrow\bullet \;\; \bullet\leftarrow$ or $\leftarrow\bullet \;\; \bullet\rightarrow$ $\Rightarrow a_t$ is $(-)$, ② If $+\bullet \;\; \bullet+$ or $-\bullet \;\; \bullet-$ $\Rightarrow a_t$ is $(+)$

Faraday's Law:

$$e(t) = N\frac{d\phi(t)}{dt} \Rightarrow E = N(j\omega)\Phi \left\{\begin{matrix} E_1 = N_1(j\omega)\Phi_c \\ E_2 = N_2(j\omega)\Phi_c \end{matrix}\right\} \Rightarrow \frac{E_1}{E_2} = \frac{N_1}{N_2} = a_t \text{ (turns ratio)}$$

or

$$\frac{E_1}{N_1} = \frac{E_2}{N_2}$$

$$\therefore \quad E_1 = \left(\frac{N_1}{N_2}\right)E_2 = a_t E_2$$

&

$$I_1 = \left(\frac{N_2}{N_1}\right)I_2 = \frac{I_2}{a_t}$$

Complex power

$$S_1 = E_1 I_1^* = (a_t E_2)\left(\frac{I_2}{a_t}\right)^* = E_2 I_2^* = S_2$$

,

$$Z_2 = \frac{E_2}{I_2}$$

This impedance, when measured from winding 1, is

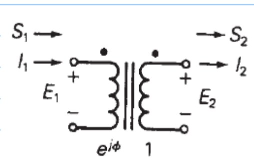
$$Z_2' = \frac{E_1}{I_1} = \frac{a_t E_2}{I_2/a_t} = a_t^2 Z_2 = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

Read Ex. 3.1

Single-phase,

Phase-shifting

transformer



$$a_t = e^{j\phi}$$

$$E_1 = a_t E_2 = e^{j\phi}E_2$$

$$I_1 = \frac{I_2}{a_t^*} = e^{j\phi}I_2$$

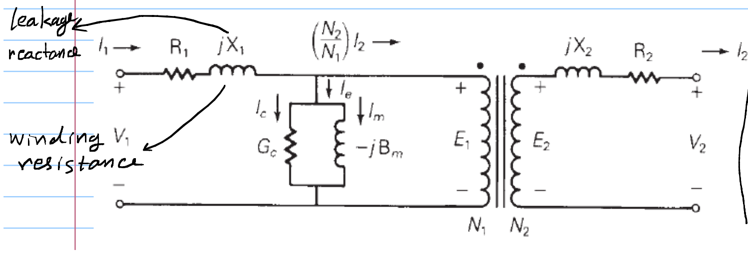
$$S_1 = S_2$$

$$Z_2' = Z_2$$

3.2 Equivalent Circuits for Practical Transformers:

Practical vs ideal transformers:

1. The windings have resistance.
2. The core permeability μ_c is finite.
3. The magnetic flux is not entirely confined to the core.
4. There are real and reactive power losses in the core.



When μ_c is finite,

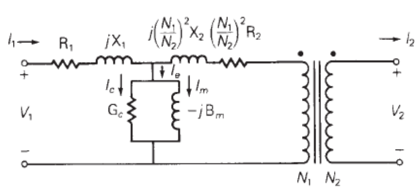
$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = \frac{\mathrm{R}_c}{N_1}\Phi_c = \frac{\mathrm{R}_c}{N_1}\left(\frac{E_1}{j\omega N_1}\right) = \overbrace{-j\left(\frac{\mathrm{R}_c}{\omega N_1^2}\right)E_1}^{\text{magnetizing current, } I_m}$$

Where $\mathrm{B}_m = \left(\frac{\mathrm{R}_c}{\omega N_1^2}\right)$ mhos.

When includin the core lose current I_c:

$$I_1 - \left(\frac{N_2}{N_1}\right) I_2 = \underbrace{I_c + I_m}_{I_e} = \underbrace{(\mathrm{G}_c - j\mathrm{B}_m)}_{Y} E_1$$

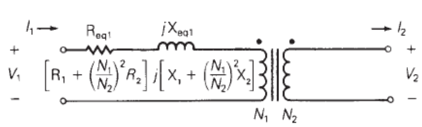
(exciting current)



(a) R_2 and X_2 are referred to winding 1

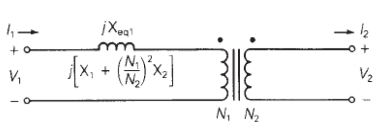
The following are not represented by the equivalent circuit

1. Saturation
2. Inrush current
3. Nonsinusoidal exciting current
4. Surge phenomena



(b) Neglecting exciting current

→ $I_e < 5\%$ of I_1



(c) Neglecting exciting current and I^2R winding loss

→ when $S > 500$ KVA,

$R_{eq} \ll X_{eq}$

3.3 The Per-Unit System:

V, I, S and Z can be expressed in pu or percentage rather than the actual values.

* Advantages:
  1- Simplified transformer equivalent circuits.
  2- Pu values are in a small range.

Solve Ex. 3.3

$$\text{per-unit quantity} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

By convention, we adopt the following two rules for base quantities:

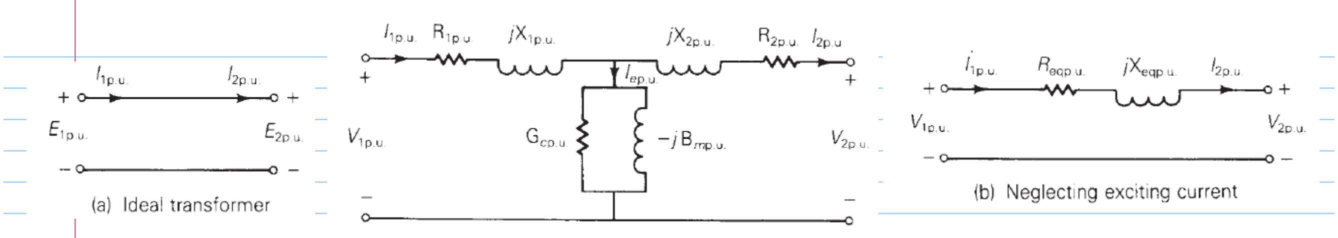
1. The value of $S_{base1\phi}$ is the same for the entire power system of concern.
2. The ratio of the voltage bases on either side of a transformer is selected to be the same as the ratio of the transformer voltage ratings.

$$P_{base1\phi} = Q_{base1\phi} = S_{base1\phi}$$

$$I_{base} = \frac{S_{base1\phi}}{V_{baseLN}}$$

$$Z_{base} = R_{base} = X_{base} = \frac{V_{baseLN}}{I_{base}} = \frac{V_{baseLN}^2}{S_{base1\phi}}$$

$$Y_{base} = G_{base} = B_{base} = \frac{1}{Z_{base}}$$



(a) Ideal transformer
(b) Neglecting exciting current
(c) Complete representation

If the pu values on the device nameplat different from the power system base values, then

$$Z_{p.u.new} = \frac{Z_{actual}}{Z_{basenew}} = \frac{Z_{p.u.old} Z_{baseold}}{Z_{basenew}} \quad \text{or} \quad Z_{p.u.new} = Z_{p.u.old}\left(\frac{V_{baseold}}{V_{basenew}}\right)^2\left(\frac{S_{basenew}}{S_{baseold}}\right)$$

Solve Ex. 3.4

In 3ϕ system, convert Δ-connected loads into Y-connected load.

$$S_{base1\phi} = \frac{S_{base3\phi}}{3}$$

$$V_{baseLN} = \frac{V_{baseLL}}{\sqrt{3}}$$

$$S_{base3\phi} = P_{base3\phi} = Q_{base3\phi}$$

$$I_{base} = \frac{S_{base1\phi}}{V_{baseLN}} = \frac{S_{base3\phi}}{\sqrt{3}V_{baseLL}}$$

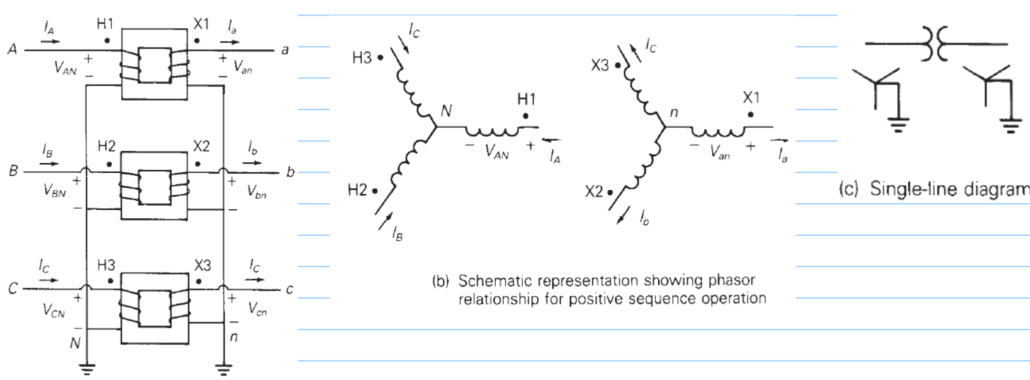
Read Ex. 3.5

$$Z_{base} = \frac{V_{baseLN}}{I_{base}} = \frac{V^2_{baseLN}}{S_{base1\phi}} = \frac{V^2_{baseLL}}{S_{base3\phi}}$$

$$R_{base} = X_{base} = Z_{base} = \frac{1}{Y_{base}}$$

3.4 Three-phase Transformer Connections and Phase Shift:

* Three-phase two-winding Y-Y transformer bank

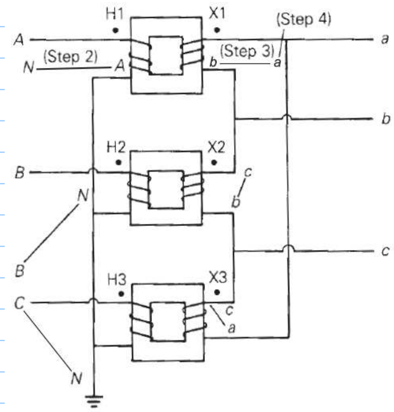


(a) Core and coil arrangements

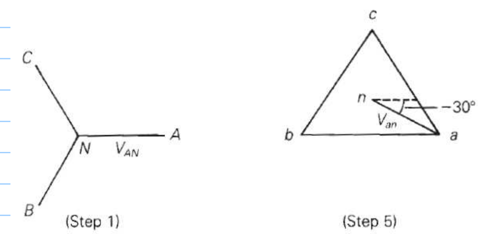
(b) Schematic representation showing phasor relationship for positive sequence operation

(c) Single-line diagram

* Three-phase two-winding Y-Δ transformer bank



(a) Core and coil arrangement

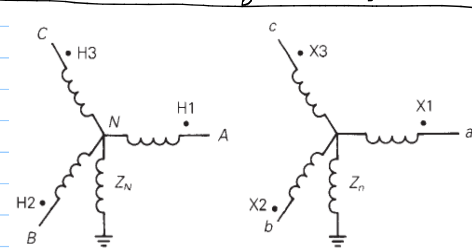


(b) Positive-sequence phasor diagram

3.5 P.U. Equivalent Circuits of Balanced 3Ø Two-winding Transformers:

By convention, we adopt the following two rules for selecting base quantities:

1. A common S_{base} is selected for both the H and X terminals.
2. The ratio of the voltage bases V_{baseH}/V_{baseX} is selected to be equal to the ratio of the rated line-to-line voltages $V_{ratedHLL}/V_{ratedXLL}$.



(a) Schematic representation

(b) Per-unit equivalent circuit for balanced three-phase operation

The per-unit equivalent circuit of the Y–Δ transformer, shown in Figure 3.17(b), includes a phase shift. For the American standard, the positive-sequence voltages and currents on the high-voltage side of the Y Δ transformer lead the corresponding quantities on the low-voltage side by 30°.

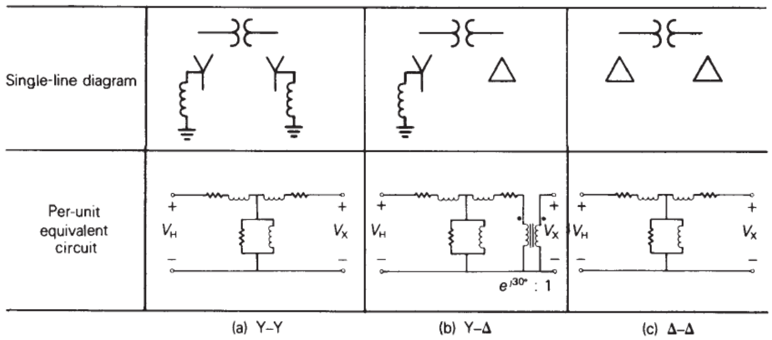


FIGURE 3.17 Per-unit equivalent circuits of practical Y–Y, Y–Δ, and Δ–Δ transformers for balanced three-phase operation

Read Ex.3.7

Solve Ex.3.8

TABLE A.2

Typical transformer leakage reactances

Rating of Highest Voltage Winding kV	BIL of Highest Voltage Winding kV	Leakage Reactance per unit*
Distribution Transformers		
2.4	30	0.023 0.049
4.8	60	0.023 0.049
7.2	75	0.026 0.051
12	95	0.026 0.051
23	150	0.052 0.055
34.5	200	0.052 0.055
46	250	0.057 0.063
69	350	0.065 0.067
Power Transformers 10 MVA and Below		
8.7	110	0.050 0.058
25	150	0.055 0.058
34.5	200	0.060 0.065
46	250	0.060 0.070
69	350	0.070 0.075
92	450	0.070 0.085
115	550	0.075 0.100
138	650	0.080 0.105
161	750	0.085 0.011

Power Transformers Above 10 MVA		Self-Cooled or Forced-Air-Cooled		Forced-Oil-Cooled	
8.7	110	0.050	0.063	0.082	0.105
34.5	200	0.055	0.075	0.090	0.128
46	250	0.057	0.085	0.095	0.143
69	350	0.063	0.095	0.103	0.158
92	450	0.060	0.118	0.105	0.180
115	550	0.065	0.135	0.107	0.195
138	650	0.070	0.140	0.117	0.245
161	750	0.075	0.150	0.125	0.250
230	900	0.070	0.160	0.120	0.270
345	1300	0.080	0.170	0.130	0.280
500	1550	0.100	0.200	0.160	0.340
765		0.110	0.210	0.190	0.350

*Per-unit reactances are based on the transformer rating

3.6 Three-winding Transformers:

$$\left.\begin{aligned} N_1 I_1 &= N_2 I_2 + N_3 I_3 \\ \frac{E_1}{N_1} &= \frac{E_2}{N_2} = \frac{E_3}{N_3} \end{aligned}\right\} \text{Actual values}$$

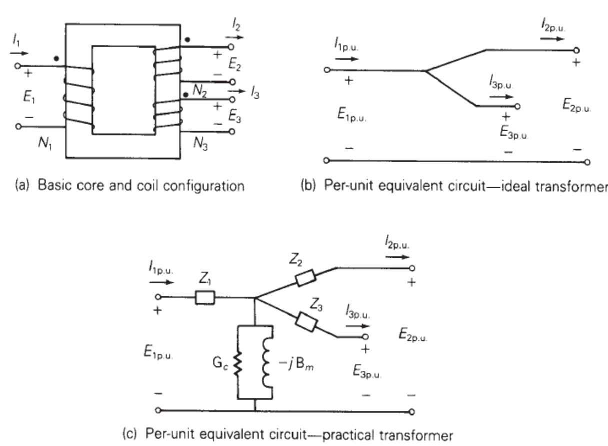


FIGURE 3.20 Single-phase three-winding transformer

$$\left.\begin{aligned} I_{1\text{p.u.}} &= I_{2\text{p.u.}} + I_{3\text{p.u.}} \\ E_{1\text{p.u.}} &= E_{2\text{p.u.}} = E_{3\text{p.u.}} \end{aligned}\right\} \text{p.u.}$$

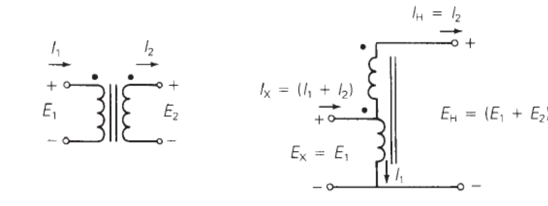
Read Ex. 3.9 & Ex. 3.10

3.7 Autotransformers:

* Coupled magnetically and electrically.
* Has smaller $X_{eq(p.u)}$
* Has lower losses.
* More economic.

Read Ex. 3.11

FIGURE 3.23
Ideal single-phase transformers



(a) Two-winding transformer (b) Autotransformer

3.8 Transformers with Off-Nominal Turns Ratios:

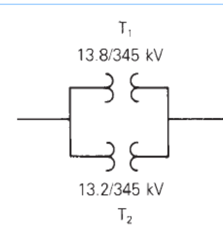
For T_1 : $a_{t(T_1)} = \frac{345}{13.8}$

For T_2 : $a_{t(T_2)} = \frac{345}{13.2}$

If $V_{baseH} = 345kV$, what V_{baseX} ?

13.8kV or 13.2kV

FIGURE 3.24
Two transformers connected in parallel



$V_{1\text{rated}} = a_t V_{2\text{rated}}$, $V_{\text{base}1} = b V_{\text{base}2}$

$$c = \frac{a_t}{b} \implies V_{1\text{rated}} = b\left(\frac{a_t}{b}\right) V_{2\text{rated}} = bc V_{2\text{rated}}$$

(Two transformers)

Alternatively: (Two-port Networks)

$$\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

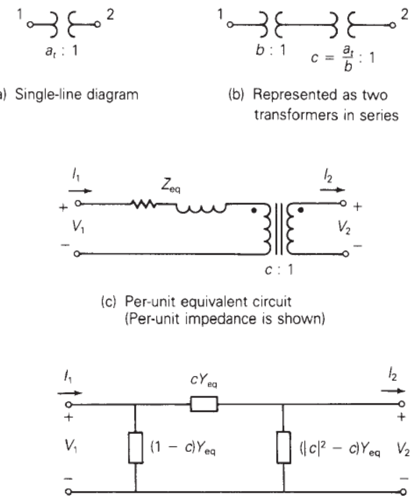
$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{1}{Z_{eq}} = Y_{eq}$$

$$Y_{22} = \frac{-I_2}{V_2}\bigg|_{V_1=0} = \frac{1}{Z_{eq}/|c|^2} = |c|^2 Y_{eq}$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0} = \frac{-cV_2/Z_{eq}}{V_2} = -cY_{eq}$$

$$Y_{21} = \frac{-I_2}{V_1}\bigg|_{V_2=0} = \frac{-c^* I_1}{V_1} = -c^* Y_{eq}$$

Solve Ex. 3.12 & 3.13



(a) Single-line diagram

(b) Represented as two transformers in series

(c) Per-unit equivalent circuit (Per-unit impedance is shown)

(d) π circuit representation for real c $\left(\text{Per-unit admittances are shown; } Y_{eq} = \frac{1}{Z_{eq}}\right)$